RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2014

THIRD YEAR

Date : 18/12/2014 **PHYSICS (Honours)**

Time : 11 am – 1 pm **Paper :** V (Gr. A) Full Marks : 50

Unit – I

(*Answer* ***any three*** *questions*) [3×10]

1. a) i) A sphere rolls without slipping on a smooth horizontal surface. What type of constraint is it subjected to? [2]

   ii) Use D'Alember's principle to set up the equation of motion of a plane pendulum of constant length. [2]

   b) An N-particle constrained system is described by f generalised coordinates q_1, q_2, ..., q_f. Express the kinetic energy of the system T as a function of q's and $\dot{q}$'s$\left(\dot{q} = \frac{dq}{dt}\right)$. If the system is seleronomic and the potential V = V(q's) show that the Hamiltonian H is given by H = T+V. [6]

2. a) A system is described by the Lagrangian $L(q, \dot{q})$ given by,

$$L(q,\dot{q}) = a\dot{q}^2 + bq^2 + 3q^2\dot{q}$$

   Show that this gives the same equation of motion as is obtained by the Lagrangian

$$L_0(q,\dot{q}) = a\dot{q}^2 + bq^2 .$$

   Explain. Do they give the same energy integral? [3]

   b) Consider the Lagrangian $L(\vec{r},\vec{v}) = \vec{v}^2 + \vec{r}.\vec{v} + \vec{r}^2, \left(\vec{v} = \frac{d\vec{r}}{dt}\right)$. Is the total angular momentum conserved? [2]

   c) Two particles of masses m_1 and m_2, radius vector $\vec{r}_1$ and $\vec{r}_2$, respectively interact via a potential $V\left(|\vec{r}_1 - \vec{r}_2|\right)$. Show that the Lagrangian of the system in terms of the centre-of-mass coordinate $\vec{R}$ and relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$ is

$$L(\vec{R}.\dot{\vec{R}},\vec{r},\dot{\vec{r}}) = \frac{1}{2}M\dot{\vec{R}}^2 + \frac{1}{2}\mu\dot{\vec{r}}^2 - v(r)$$

   when $M = m_1 + m_2$ and $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass

   Deduce the equations of motion in $\overline{R}$ and $\overline{r}$ variables. Find out the cyclic coordinates, and hence deduce the relevant conservation principles. [5]

3. a) A particle of mass m is attracted to fixed point O by an inverse square force $F = -\frac{GMm}{r^2}$. Justify the use of plane polar coordinates and use these to find the Hamiltoniam and hence Hamilton's equation of motion. Identify any conserved quantity, if possible. [5]

   b) Given that $\vec{L} = \vec{r} \times \vec{p}$, show that $[L_x, L_y]_{P.B} = L_z$. [3]

   c) Show that for a free particle of mass (m) moving with a velocity (v), the variable $(x - vt)$ is a constant of motion. [Assume the relevant formula, $\frac{dA}{dt} = [A,H]_{P.B} + \frac{\partial A}{\partial t}$] [2]

4. a) Set up Euler's equation of motion for a symmetrical top in a force-free space. (choose the symmetry axis to the third principal axis). Show that for this type of motion (i) the angular

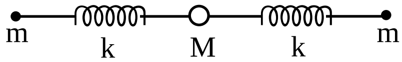
velocity vector $\vec{\omega}$ precesses about the symmetry axis with a constant angular speed and (ii) the total kinetic energy T is constant. [5]

b) Use the principal of least action for a haloxamic system with Lagrangian L to derive Lagrange's equations of motion. Hence show that the Lagrangian L' defined by

$$L' = L + \frac{d}{dt} f(q,t),$$

when f(q,t) is a function of generalised coordinates and time, gives the same equations of motion as L. [5]

5. Consider a longitudinal motion of a symmetric triatomic molecular shown below :



a) Find the normal frequencies of the system. [6]

b) Interpret the nature of the oscillations. [4]

Unit – II

(*Answer **any two** questions*) [2×10]

6. a) Write down the Lorentz transformation between two inertial frames which are in relative uniform motion with velocity v along the x axis. Show that the speed of light in vacuum remains the same from these two frames. [5]

b) An astronaut wants to go to a star 15 light years away. The rocket accelerates quickly in a negligible time and then moves at a uniform velocity. With what velocity must the rocket move relative to the earth if the astronaut is to reach there in one year, as measured by clocks at rest on the rocket? Deduce the formula you use here. [5]

7. a) Define the 4-vector velocity and 4-force in terms of the proper time interval $d\tau$. Show that the scalar product of 4-force and 4-velocity is zero. Hence find the relationship between the energy and momentum of a relativistic particle. [5]

b) A relativistic particle of rest mass m is moving with velocity v. Show that its kinetic energy can be given as $\frac{\gamma^2}{\gamma+1} mv^2$, where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$. Find the non-relativistic form of the kinetic energy. [5]

8. a) How fast must you be driving your car to see a red light signal as green? Take the wave lengths of red and green lights as 630nm and 540 nm respectively. Deduce the formula you employ here. [5]

b) Define space-like light like and time-like intervals. Show that two events connected by space-like intervals may be made to appear simultaneous in a suitable Lorentz frame. [5]

9. a) Explain the null results of Michenson-Morley's experiment and show how they are consistent with the fundamental postulates of relativity. [4]

b) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under Lorentz transformation. [4]

c) Show that $U_\mu U_\mu = -c^2$ when U_μ is a component of four-velocity, U. [2]

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